Compeng 3SK3

Project 1  
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The pseudocode and simulation program are included in the MATLAB file.

4)

a) No, you cannot because the IEEE-754 32-bit floating point number has an inherent max precision of 6-9 significant decimal digits. Therefore, at some point, 1/N, will become too small to be representable by a 32-bit floating point number.

b) Using an IEEE 32-bit floating point number, the minimum N value such that 1/N becomes too small to be representable is N = 2^128, as any integer greater than or equal to 2^128 is rounded to infinity. The minimum N value such that 1/(N - 1) - 1/N becomes too small to be representable is N = 2^24 + 4. When I plugged this value - 1 into the equation represented in MATLAB, I got 0.0000000000000071054273576010018587112426757812, so 2^24 + 3 is still representable. This is the code I used:

N = 2^24 + 3;

test = vpa(single(single(1.0) / single(N - 1) - single(1.0) / single(N)))

The vpa() does not change the actual value of the answer, without vpa() I get 7.1054e-15.

The minimum N value such that 1/N is smaller than machine precision is N = 2^23 + 1 because there are only 23 bits of mantissa in a 32-bit float. The minimum value of N such that 1/(N - 1) - 1/N is smaller than machine precision is N = 2898, which gives a result of 0.00000011912197805941104888916015625, and machine precision is 0.00000011920928955078125. I got this value by doing a linear search in MATLAB for values of N between 2^11 and 2^12.

c)

Chart, line chart

Description automatically generated

d) It does not. I only achieved a precision of 1.9047e-09 with an N value of 2^30. One way to improve the precision of the algorithm would be to use [arbitrary-precision arithmetic](https://en.wikipedia.org/wiki/Arbitrary-precision_arithmetic) or use a [second-order "iterative Kahan–Babuška algorithm"](https://en.wikipedia.org/wiki/Kahan_summation_algorithm#Further_enhancements) to increase the accuracy some more.